# 4th Annual Lexington Mathematical Tournament
# Theme Round

March 30th, 2013

## 1 Apple Pi and Other Desserts

*"In order to make an apple pie from scratch, you must first create the universe."* – Carl Sagan

1. Surya decides to sell gourmet cookies at LMT. If he sells them for 25 cents each, he sells 36 cookies. For every 4 cents he raises the price of each cookie, he sells 3 fewer cookies. What is the smallest possible price, in cents, that Surya's jealous friends can change each cookie to so that Surya has no customers?

2. Three French sorbets and four Italian gelatos cost 144 dollars. Six Italian gelatos and nine Florida sundaes cost 117 dollars. Seven French sorbets and 1 Florida sundae cost 229 dollars. If Arul wants to buy one of each type of ice cream, how much will it cost him?

3. We call a number *delicious* if it has exactly 4 factors and is divisible by 2. How many numbers less than 99 are *delicious*?

4. Charlie has 6 apple pies that form a ring. He starts with the first pie and puts on 10 scoops of whipped cream. Then, he moves to the next pie and, with a $\frac{1}{2}$ chance for each case, puts on either one less or one more scoop of whipped cream than the amount he put on the previous pie. He continues this process until he puts some whipped cream on the sixth pie. What is the probability that the number of scoops of whipped cream on the sixth pie differs from that on the first pie by exactly 1?

5. Hao has 32 ounces of pure lemonade in his juice box. On the first day, he drinks an ounce and then refills the juice box with an ounce of Hater-ade and mixes the mixture thoroughly. On the second day, he drinks 2 oz of the juice and then refills the juice box with 2 oz of Hater-ade. On the third day, he drinks 3 oz and then refills the juice box with 3 oz of Hater-ade, and so on. Hao stops when the juice box has no more lemonade in it. How many ounces of Hater-ade did Hao drink?

## 2 Game Theory

*"You have to learn the rules of the game. And then you have to play better than anyone else."*
– Albert Einstein

6. Three standard six-sided dice are rolled and the values on the top faces are added. Let $p$ be the probability of getting a total sum of 17 and $q$ be the probability of getting a total sum of 18. Find $p/q$.

7. Bill, Bob, and Ben write their favorite numbers on a sheet of paper. Bob points out that each number has its digits sum to 4, Bill points out that they are all divisible by the same prime number greater than 10, and Ben points out that none of them have 0's. Find the sum of Bill's, Bob's, and Ben's favorite numbers, given no two of them have the same favorite number.

8. Sara has a $3 \times 3$ square tiled with alternating black and white colors (like a checkerboard). When she chooses a square, that square and the squares that share an edge with it switch colors. What is the minimum number of squares Sara needs to choose in order to cover the board with one color?

9. A checker piece of radius 1 is placed at an arbitrary location on an infinitely large checkerboard of $1 \times 1$ squares . The checker piece covers $n$ squares, either partially or completely. How many possible values for $n$ are there?

10. In a game of laser tag, a robot is in the center of a rectangular room with dimensions 6 meters by 8 meters. The robot has two perpendicular arms that can fire lasers straight outwards. The robot can fire the two lasers so long as they would hit the same wall if nothing blocked them. If a person is standing at a random location inside the room, what is the probability that she can be hit by a laser? The lasers do not bounce off walls.

# 3 Dream Jobs

"*Choose a job you love, and you will never have to work a day in your life.*" – Confucius

11. Darwin has a fruit stand at the market. Frank purchases 5 apples, and after a 5% tax, the price is $2.50. Next, Rohil buys 3 apples. Assuming that everyone pays the 5% tax, how much did Rohil have to pay?

12. In architecture class, Professor Radian wants to build a bridge over his circular pond with radius 4. He randomly chooses two points A and B on the circumference of the pond to be the endpoints of his new bridge. What is the probability that the length of the bridge is greater than $4\sqrt{3}$?

13. Track stars Noah and Jonah run around a circular track. It takes Noah 3 minutes 20 seconds to run around the track, while it takes Jonah 3 minutes 45 seconds to run around the track. They start running around the track from the same spot going in the same direction. In how many minutes will the two of them be in the same spot along the track?

14. Dan the Detective needs to decrypt a secret message intercepted from a clan of first graders. There are only 6 letters in the clan's alphabet, and Dan knows that the encryption code used by the clan is created by replacing every letter with another. For example, one encryption code may be $A \to B \to C \to D \to E \to F \to A$, where $A \to B$ means that every instance of $A$ is replaced by a $B$. Note that a letter cannot be replaced by itself and no letter can replace multiple other letters. Given these conditions, how many different encryption codes can exist?

15. The High Guardians of LHS have been assigned to protect a new, high priority room. A High Guardian's field of vision has a 360° range so that it can see everything not directly blocked by a wall – in other words, one cannot see around walls – but once placed in the room, a High Guardian cannot move from that position. The room has 8 straight walls along its boundary and needs to be guarded so that every point in the entire room is in the field of vision of at least one High Guardian. What is the minimum number of High Guardians necessary for this task, no matter how the room is shaped?